

Quantum analog of channeled electron trajectories in periodic magnetic and electric fields

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Abstract

We calculate the quantum states corresponding to the drifting and channeled classical orbits in a two-dimensional electron gas (2DEG) with strong magnetic and electric modulations along one spatial direction, x . The channeled states carry high, concentrated currents along the y axis, and are confined in an effective potential well. The quantum and the classical states are compared.

The classical equations of motion of an electron in a plane, in the presence of a strong unidirectional magnetic or electric modulation, plus an external perpendicular magnetic field, have two types of solutions: drifting and channeled orbits. The former are slow, with self-intersecting cyclotron loops, while the latter are fast, snaking around the lines of zero magnetic field in the magnetic modulation and in the vicinity of the potential minima in the electric one. The channeled orbits are in the classical explanation responsible for the positive magneto resistance observed for low magnetic fields [1,2]. Recent experiments show a giant effect in strong magnetic modulations, with estimated amplitude 0.6 T and period 500 nm [3]. Such conditions seem to require quantum mechanical calculations. Quantum states in a linearly varying magnetic field have been calculated in [4].

We consider the 2DEG, in the (x, y) plane, in a periodic magnetic field which has the form $\mathbf{B}(x) = (B_0 + B_1 \cos Kx)\mathbf{e}_z$. The case with zero average field has been recently discussed [5]. Here we always keep a finite B_0 , denoting by $s = B_1/B_0$ the modulation strength. The Hamiltonian is $H = (\mathbf{p} + e\mathbf{A}(x))^2/2m$, where, in the Landau gauge, $\mathbf{A}(x) = (0, B_0x + (s/K)\sin Kx)$. Classically, the possible trajectories are determined by two conserved quantities, the energy E and the canonical momentum in y -direction, $p_y \equiv -X_0\omega_0m$. Energy conservation, $E = mv_x^2/2 + V(X_0; x)$, can be used to calculate the velocity component $v_x(X_0, E; x)$ [2]. Since $B_0 \neq 0$ the electron is bound in x -direction, so that the orbit has turning points x_{\pm} given by the condition $v_x(X_0, E; x_{\pm}) = 0$. Quantum mechanically, translational invariance in y -direction implies wave functions of the form $L_y^{-1/2} \exp(-iX_0y/l_0^2)\psi_{n,X_0}(x)$, where L_y is a normalization length, $n = 0, 1, 2, \dots$, and X_0 is a center coordinate. We denote by $l_0 = (\hbar/eB_0)^{1/2}$ the magnetic length and by $\omega_0 = eB_0/m$

the cyclotron frequency, corresponding to the average field. The electron effective mass is that of GaAs, $m = 0.067m_0$, and we assume spin degeneracy.

The reduced wave functions $\psi_{n,X_0}(x)$ are the solutions of a one-dimensional Schrödinger equation with the effective potential $V(X_0; x) = \hbar\omega_0(x - X_0 + \frac{s}{K} \sin Kx)^2/2l_0^2$. For a fixed X_0 it has extrema at the positions where the magnetic field is zero, i. e. at the roots of $1 + s \cos Kx = 0$, and additional minima at the points where $V(X_0; x) = 0$. We use the Landau wave functions (solutions for $s = 0$) as the basis of our Hilbert space to obtain the eigenstates for $s \neq 0$ by numerical diagonalization. The necessary basis size is here between 150-300 Landau levels. We can distinguish two situations: $s < 1$, when the magnetic field $B(x)$ always points up, and $s > 1$, when it has alternating sign. For $s < 1$ the energy spectra consist in more or less perturbed Landau levels reorganized in periodic Landau bands, E_{n,X_0} , with the first Brillouin zone defined by $0 \leq X_0 < a \equiv 2\pi/K$. The extension of the perturbed wave functions in the x direction is about $2R_c$, with $R_c = l_0\sqrt{2n+1}$ the cyclotron radius in the average field [6].

For $s > 1$, the energy spectra are more complicated, with strong overlap of the bands, Fig. 1. To understand the nature of the states we plot in Fig. 2 (a) four characteristic states, indicated with dots in Fig. 1, having the quantum numbers $n = 6, 7, 43$, and 44 , and the same center coordinate $X_0 = 0.1a$, so that all of them have the same effective potential. Corresponding classical orbits are shown in Fig. 2 (b). There are eight classical orbits, since for each of the four selected energies there are *two* solutions. We show only the four orbits with energies E_{6,X_0} and E_{43,X_0} , which are similar to those with E_{7,X_0} and E_{44,X_0} , respectively. Three cycles of the classical orbits are plotted in each case.

It is clearly seen, that the wide-spread quantum states, $n = 7, 44$, correspond to the drifting orbits of the classical picture. The states $n = 6, 43$, localized within shallow valleys of $V(X_0; x)$, correspond to channeled orbits snaking along the lines with $B(x) = 0$ [4]. The state with $n = 6$ is the lowest (no node) quantized state bound near $x/a = -2/3$, and that with $n = 43$ the second (one node) near $x/a = 5/3$. The energy spacing between the quantum states depends on the width of the potential well, hence it is larger for the ‘localized’ channeled states than for the ‘extended’ drifting ones. Also, hybridization effects forbid degeneracy (for a fixed X_0) and hence the quantum analogs of channeled and drifting orbits cannot have the same energy, unlike the classical solutions. The apparent intersections of the Landau bands are in fact anti-crossings, with extremely small gaps, due to the exponentially small spatial overlap of nearly isoenergetic states separated by large effective potential barriers. We indicated one individual band with the dotted line in Fig. 3 (a). Now the interpretation of Fig. 1 becomes clear: the drifting states give the branches with weak dispersion, and the channeled states the branches with strong dispersion, i.e. with high group velocity $\langle v_y \rangle = -(m\omega_0)^{-1}dE_{n,X_0}/dX_0$, back-folded like free electron parabolas.

The motion in x -direction being periodic, one can define a classical localization probability density, $W(x)$, by comparing the time dt , which the electron spends in the interval dx at x , with the period T : $W(x)dx = dt/T$. Due to the vanishing velocity this function diverges at x_{\pm} . In Fig. 3 we compare the quantum mechanical probability density with the classical one at the energy $E = 4$ meV, for which we have 14 states in the Brillouin zone. Due to the symmetry we display only the states with $X_0 < a/2$, indicated with dots in the relevant part of the spectrum, Fig. 3 (a). The wave functions of the first four states (with

different X_0), belonging to energy-band segments with strong dispersion, are now located in minima of different effective potentials, but are channeled around the same line $B(x) = 0$. At the chosen energy the classical trajectory is split into channeled and drifting orbits. The classical solutions merge for the states (v)-(vii), which in the quantum description belong to those regions of the spectrum where only drifting states are possible, widely spread in the whole effective potential.

For s bigger than in Fig. 1, when $V(X_0; x)$ may have more than one zero, the drifting states become more complicated. This situation and other details will be discussed in a forthcoming paper.

In the modulated systems fabricated by the deposition of metallic micro-strips of magnetic material on top of the semiconductor there is always a stress-induced electric modulation which may be shifted in phase with respect to the magnetic one [7]. In a simple sinusoidal model for the electric modulation the effective potential becomes $V(X_0; x) + U \cos Kx$, where U is the electrostatic potential amplitude. For a pure electric modulation ($B_1 = 0$), when $U/\hbar\omega_0$ is sufficiently large, the effective potential has local minima situated close to the minima of the real potential where again channeled states are captured. The physical mechanism is now a combined effect of strong electric and weak Lorentz forces and the channeled orbits vanish with increasing energy [1]. The channeled states carry positive and negative currents along the y axis which in Fig. 1 cancel each other within each unit cell. For an asymmetric situation, resulting by combining phase-shifted magnetic and electric modulations, it is possible to obtain channeled currents running only one way along the y axis, and never in the opposite direction, in a wide energy interval. Of course, in an equilibrium situation there can be no net current, and the current carried by channeled states is compensated by the current carried by the drifting states. In Fig. 4 we have chosen a magnetic modulation with $s < 1$ and an electrostatic potential $U \cos(Kx + \pi/2)$. The channeled states appear thus due to the electric modulation.

In conclusion we have discussed the quantum drifting and channeled electronic states in strong modulations and compared them with classical electron trajectories. For understanding the recent experiments [3] quantum transport calculations in this regime could be of great interest.

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FIGURES

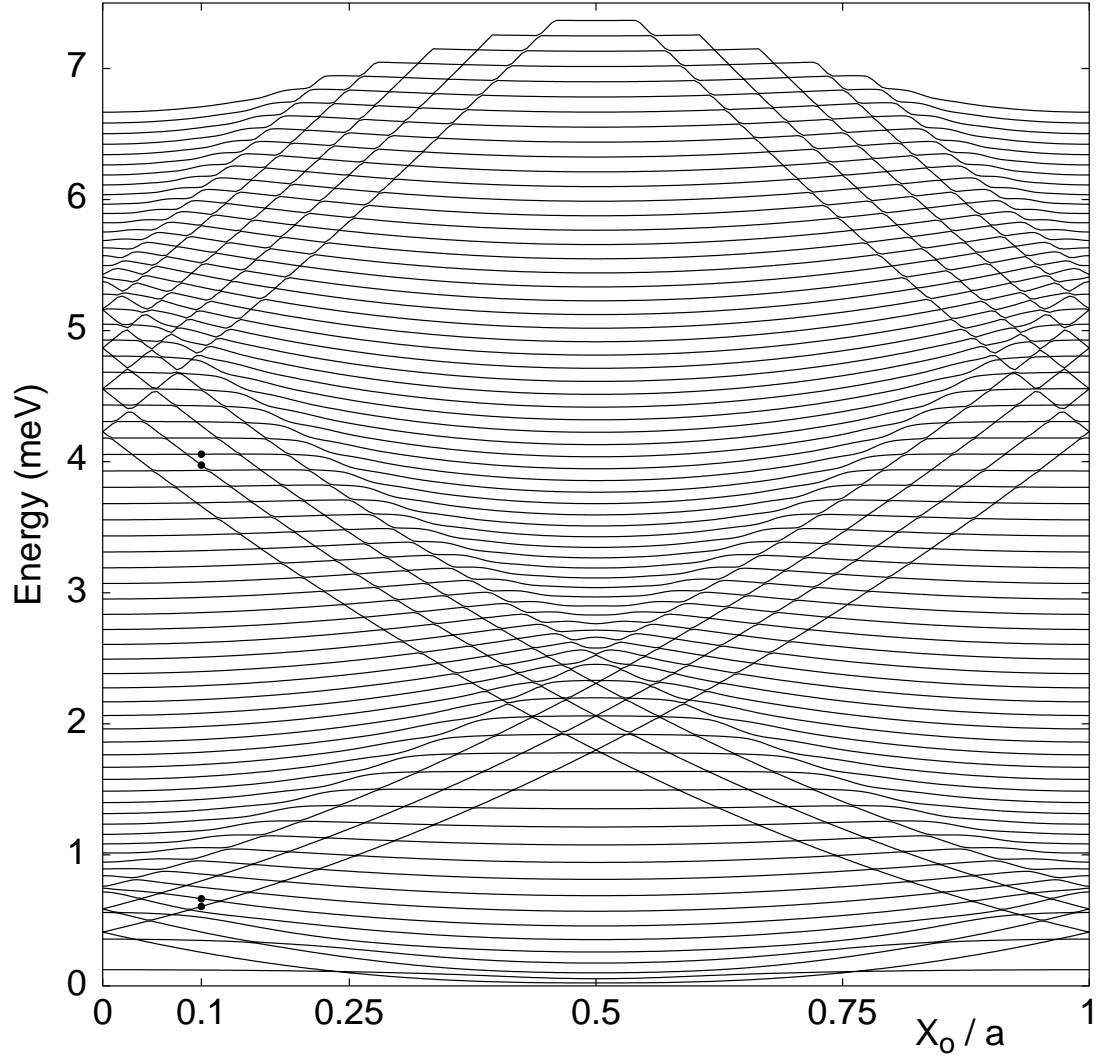


FIG. 1. Energy spectrum for $s = 2$, $B_0 = 0.05$ T, $B_1 = 0.1$ T, and $a = 800$ nm. The dots mark the states evaluated in Fig. 2.

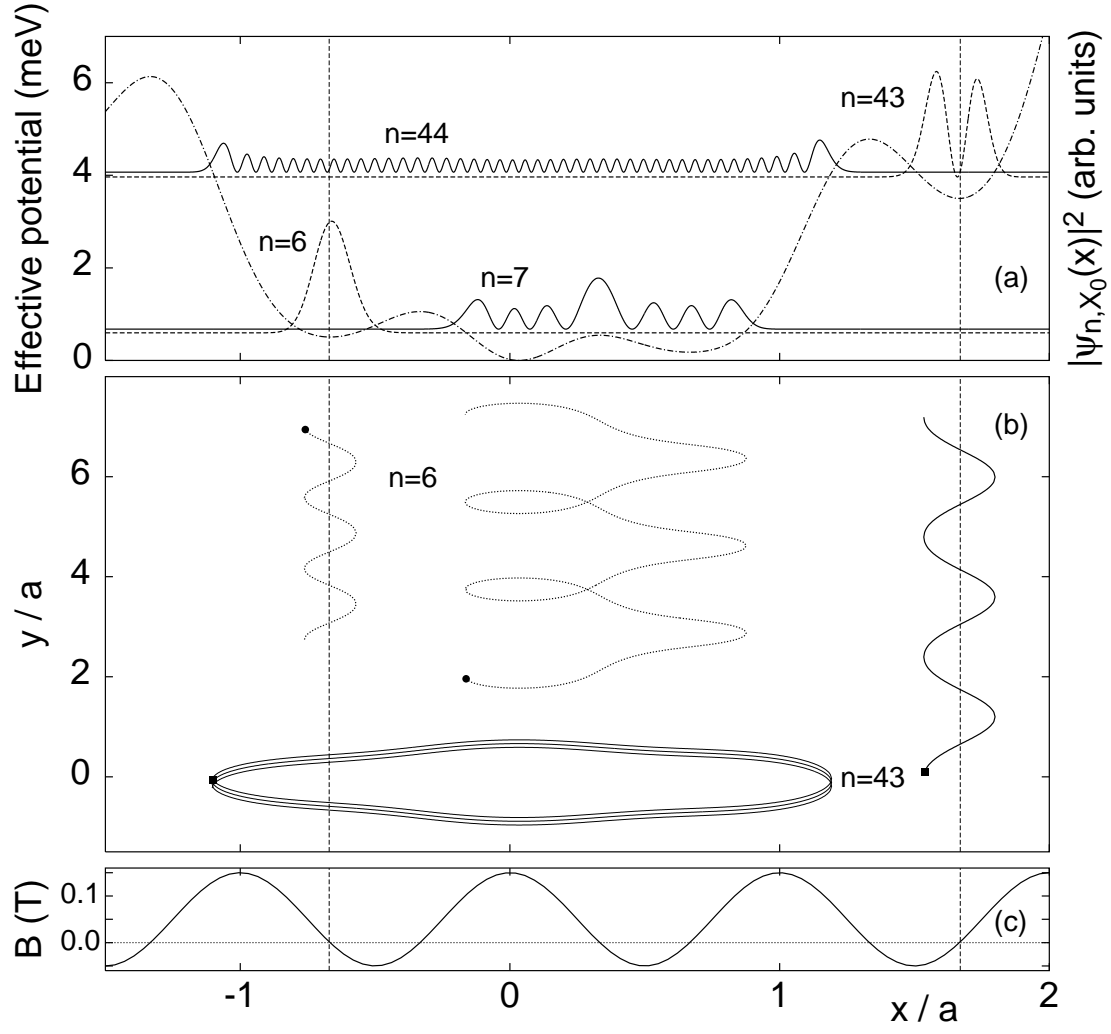


FIG. 2. Comparison of wave functions and classical trajectories for $X_0 = 0.1a$. (a) The squared modulus of the wave functions and the corresponding effective potential (dash dotted line) for the states marked by the points in Fig. 1. The eigenvalues of the states are indicated by an energetical offset (left scale). (b) Trajectories for energies corresponding to $n = 6$ and $n = 43$, beginning marked with dots. (c) The periodic magnetic field.

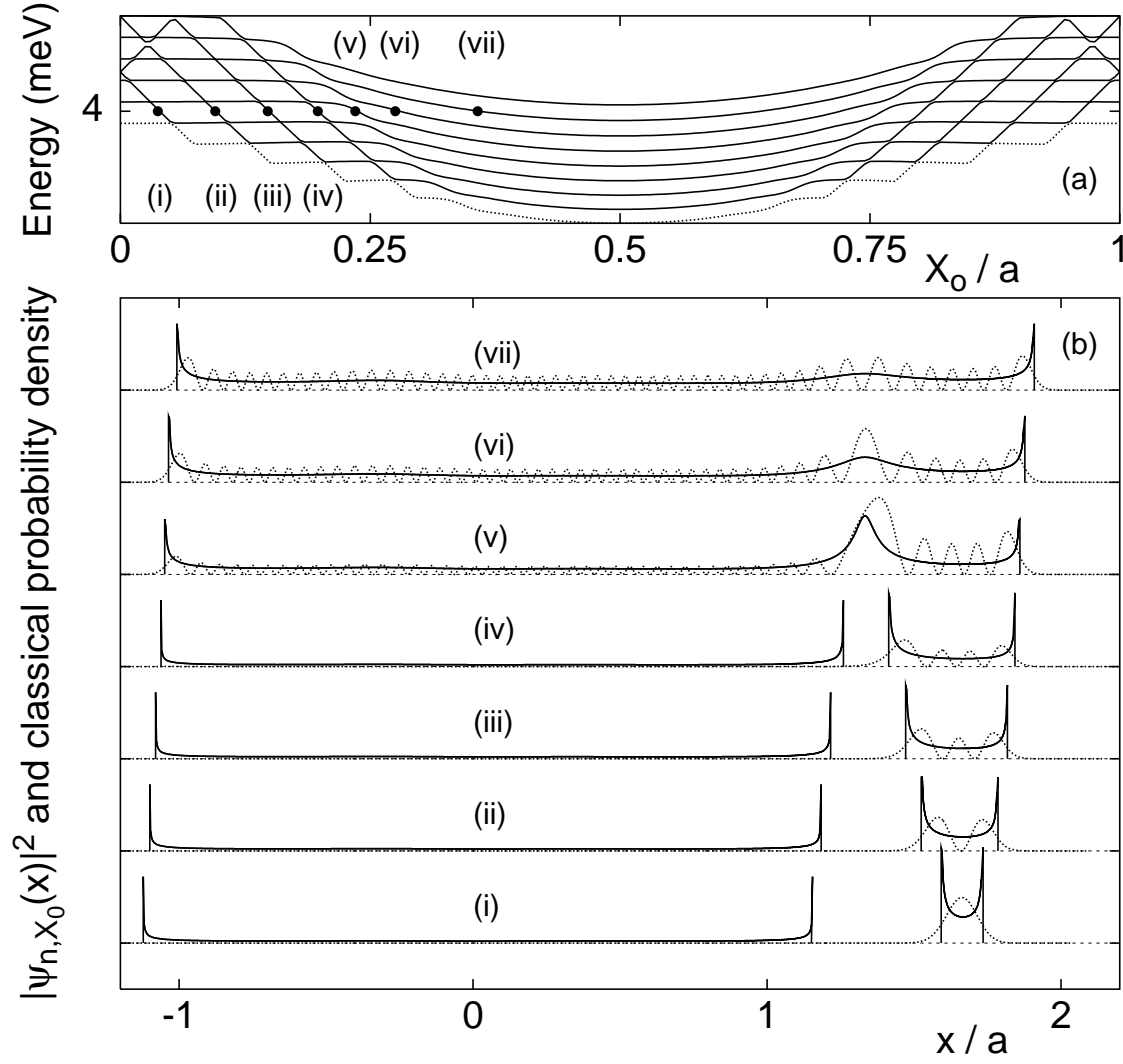


FIG. 3. (a) An extract from energy spectrum, Fig. 1. Dotted line: band $n = 41$. (b) Quantum (dotted) and classical (solid lines) probability density at fixed energy, $E = 4$ meV, for the states marked in (a), in arbitrary units. Amplitudes of plots (v), (vi), and (vii) are amplified by a factor 5.

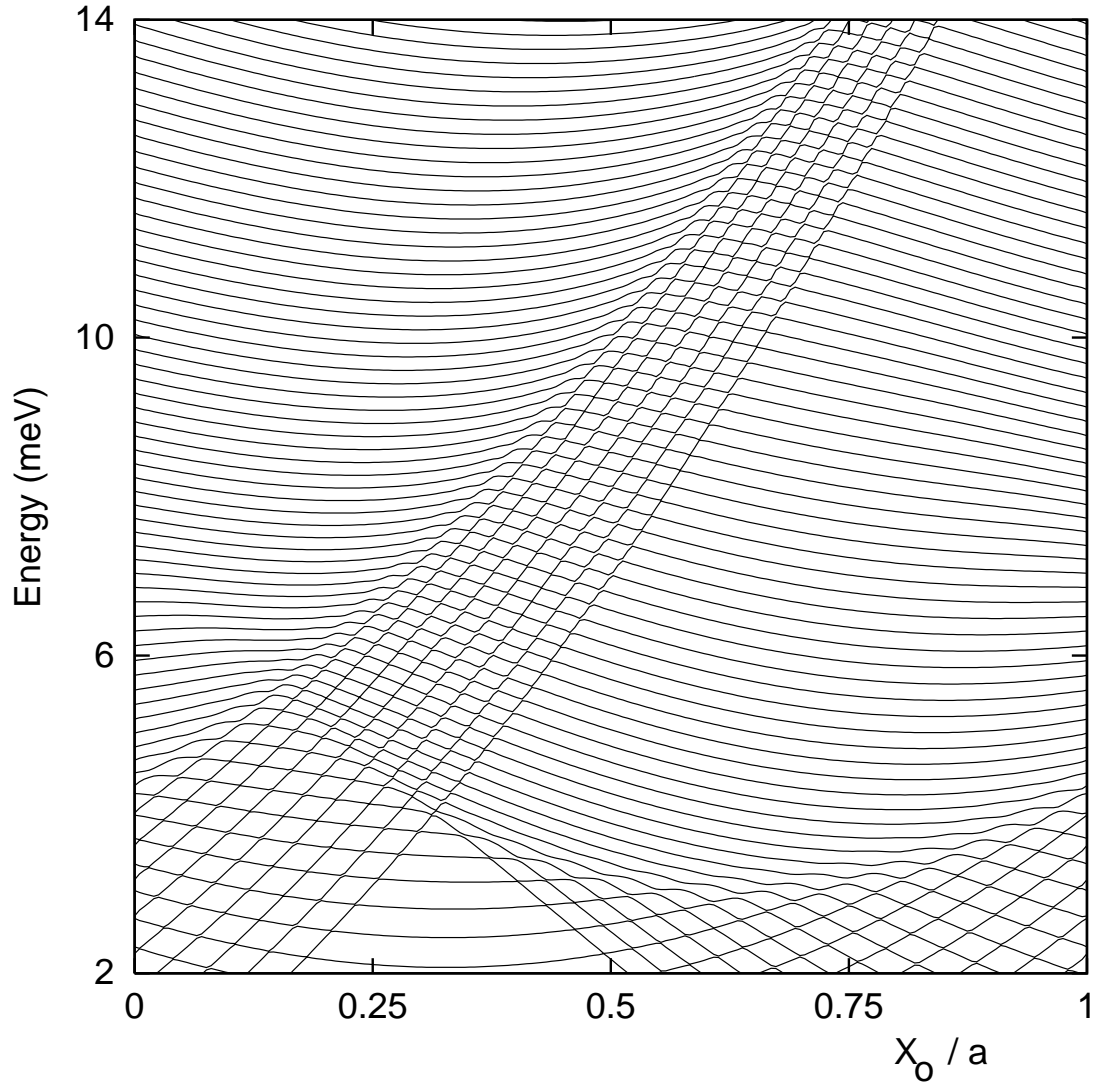


FIG. 4. Energy spectrum for combined magnetic and electric modulations, shifted with $\pi/2$. $B_0 = 0.1$ T, $B_1 = 0.08$ T, and $U = 3$ meV.